ELC 4340

POWER SYSTEMS

Lecture 13

Power Flows: Newton-Raphson Iterative Method

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Announcements

- Be reading Chapter 6, also Chapter 2.4 (Network Equations).
- HW 7 is due by October 21.

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Newton-Raphson Algorithm

- The second major power flow solution method is the Newton-Raphson algorithm
- Key idea behind Newton-Raphson is to use sequential linearization

General form of problem: Find an $\hat{x}$ such that

$$f(\hat{x}) = 0$$

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Power Flow

5. <u>Newton-Raphson Method</u>

- Iterative Solutions for Nonlinear Equations

Consider a scalar equation:

$$y = f(x)$$

Taylor series expansion of $f(x)$ about an operating point x_0 yields

$$y = f(x_0) + \left.\frac{df}{dx}\right|_{x_0}(x - x_0) + \frac{d^2f}{dx^2}\frac{(x-x_0)^2}{2!} + \cdots$$

or $$y = f(x + \Delta x) = f(x) + \frac{df}{dx}\Delta x + \cdots$$

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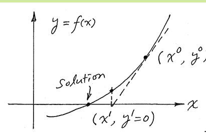
Power Flow



$$f(x^1) = f(x^0) + f'(x^0)(x^1 - x^0)$$

$$y^1 = f(x^1) \neq 0$$

In approximation,

$$0 = f(x^0) + f'(x^0)(x^1 - x^0)$$

$$\Rightarrow \quad x^1 = x^0 - \frac{f(x^0)}{f'(x^0)}$$

In iteration,

$$x^k = x^{k-1} - \frac{f(x^{k-1})}{f'(x^{k-1})}$$

Repeating the iteration, the true solution for

$$0 = f(x)$$

can be found as shown in Fig.

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Power Flow

For vector equations

$$y_1 = f_1(x_1, x_2, \cdots, x_n)$$

$$\vdots$$

$$y_n = f_n(x_1, x_2, \cdots, x_n)$$

Taylor series expansion about $(x_1^0, \cdots, x_n^0)$ yields

$$y_1 = f_1(x_1^0, \cdots, x_n^0) + \left.\frac{\partial f_1}{\partial x_1}\right|_0 \Delta x_1^0 + \cdots + \left.\frac{\partial f_1}{\partial x_n}\right|_0 \Delta x_n^0$$

$$\vdots$$

$$y_n = f_n(x_1^0, \cdots, x_n^0) + \left.\frac{\partial f_n}{\partial x_1}\right|_0 \Delta x_1^0 + \cdots + \left.\frac{\partial f_n}{\partial x_n}\right|_0 \Delta x_n^0$$

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Power Flow

$$y_1 = f_1(x_1^0, \cdots, x_n^0) + \left.\frac{\partial f_1}{\partial x_1}\right|_0 \Delta x_1^0 + \cdots + \left.\frac{\partial f_1}{\partial x_n}\right|_0 \Delta x_n^0$$

$$\vdots$$

$$y_n = f_n(x_1^0, \cdots, x_n^0) + \left.\frac{\partial f_n}{\partial x_1}\right|_0 \Delta x_1^0 + \cdots + \left.\frac{\partial f_n}{\partial x_n}\right|_0 \Delta x_n^0$$

$\Rightarrow$

$$\begin{bmatrix} y_1 - f_1^0 \\ \vdots \\ y_n - f_n^0 \end{bmatrix} = \underbrace{\begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_0 & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_0 \\ \vdots & & \\ \left.\frac{\partial f_n}{\partial x_1}\right|_0 & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_0 \end{bmatrix}}_{\text{Jacobian Matrix}} \begin{bmatrix} \Delta x_1^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix}$$

In matrix form,

$$\Delta y = J \, \Delta x$$

which is a linear (matrix) equation that we can solve using methods such as gaus elimination procedure.

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Power Flow

Ex. $f_1(x_1, x_2) = 2x_1^3 + 3x_1^2 x_2 - x_2^2 - 2 = 0$

$f_2(x_1, x_2) = x_1 x_2^2 + 2x_1^2 - 3x_2^2 + 16 = 0$

$\frac{\partial f_1}{\partial x_1} = 6x_1^2 + 6x_1 x_2$ $\qquad$ $\frac{\partial f_1}{\partial x_2} = 3x_1^2 - 2x_2$

$\frac{\partial f_2}{\partial x_1} = x_2^2 + 4x_1$ $\qquad$ $\frac{\partial f_2}{\partial x_2} = 2x_1 x_2 - 6x_2$

Let $(x_1^0, x_2^0) = (2, 2)$, initial guess

$f_1^0 = 16 + 24 - 4 - 2 = 34$

$f_2^0 = 8 + 8 - 12 + 16 = 20$

$\left.\frac{\partial f_1}{\partial x_1}\right|_0 = 24 + 24 = 48$ $\qquad$ $\left.\frac{\partial f_1}{\partial x_2}\right|_0 = 12 - 4 = 8$

$\left.\frac{\partial f_2}{\partial x_1}\right|_0 = 4 + 8 = 12$ $\qquad$ $\left.\frac{\partial f_2}{\partial x_2}\right|_0 = 8 - 12 = -4$

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Power Flow

$$y_1 - f_1^0 = \left.\frac{\partial f_1}{\partial x_1}\right|_0 \Delta x_1^0 + \left.\frac{\partial f_1}{\partial x_2}\right|_0 \Delta x_2^0$$

$$y_2 - f_2^0 = \left.\frac{\partial f_2}{\partial x_1}\right|_0 \Delta x_1^0 + \left.\frac{\partial f_2}{\partial x_2}\right|_0 \Delta x_2^0$$

$\Rightarrow$

$0 - 34 = 48 \Delta x_1^0 + 8 \Delta x_2^0 \quad \cdots (1)$

$0 - 20 = 12 \Delta x_1^0 - 4 \Delta x_2^0 \quad \cdots (2)$

(1) $+ 2 \times$ (2): $-74 = 72 \Delta x_1^0 \rightarrow \Delta x_1^0 = -1.03$

(1): $\Delta x_2^0 = 1.9$

$\Rightarrow x_1^1 = x_1^0 + \Delta x_1^0 = 2 - 1.03 = 0.97$

$x_2^1 = x_2^0 + \Delta x_2^0 = 2 + 1.9 = 3.97$

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Power Flow

2nd iteration:

$$4.3 = 28.4\,\Delta x_1^1 - 5.0\,\Delta x_2^1$$
$$13.0 = 19.1\,\Delta x_1^1 - 15.8\,\Delta x_2^1$$

$\Rightarrow \quad \Delta x_1^1 = 0.0085, \quad \Delta x_2^1 = -0.812$

$\Rightarrow \quad x_1^2 = 0.97 + 0.0085 = 0.9785$

$x_2^2 = 3.97 - 0.812 = 3.09$

In few iterations the solution converges to the true solution, (1, 3).

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Multi-Variable Example

Solve for $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ such that $\mathbf{f}(\mathbf{x}) = 0$ where

$$f_1(\mathbf{x}) = 2x_1^2 + x_2^2 - 8 = 0$$
$$f_2(\mathbf{x}) = x_1^2 - x_2^2 + x_1 x_2 - 4 = 0$$

First symbolically determine the Jacobian

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} \end{bmatrix}$$

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Multi-variable Example, cont'd

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} 4x_1 & 2x_2 \\ 2x_1 + x_2 & x_1 - 2x_2 \end{bmatrix}$$

Then

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = -\begin{bmatrix} 4x_1 & 2x_2 \\ 2x_1 + x_2 & x_1 - 2x_2 \end{bmatrix}^{-1} \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \end{bmatrix}$$

Arbitrarily guess $\mathbf{x}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}^{-1} \begin{bmatrix} -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 1.3 \end{bmatrix}$$

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Multi-variable Example, cont'd

$$\mathbf{x}^{(2)} = \begin{bmatrix} 2.1 \\ 1.3 \end{bmatrix} - \begin{bmatrix} 8.40 & 2.60 \\ 5.50 & -0.50 \end{bmatrix}^{-1} \begin{bmatrix} 2.51 \\ 1.45 \end{bmatrix} = \begin{bmatrix} 1.8284 \\ 1.2122 \end{bmatrix}$$

Each iteration we check $\|\mathbf{f}(\mathbf{x})\|$ to see if it is below our specified tolerance ε

$$\mathbf{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 0.1556 \\ 0.0900 \end{bmatrix}$$

If $\varepsilon = 0.2$ then we would be done. Otherwise we'd continue iterating.

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Power Flow

6. Power-Flow Solution by Newton-Raphson

Power equations:

$$P_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

$\Rightarrow$

$$y = f(x), \quad \text{nonlinear}: 2N \times 1$$

where

$$y = \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} P_1 \\ \vdots \\ P_N \\ \hdashline Q_1 \\ \vdots \\ Q_N \end{bmatrix} \qquad x = \begin{bmatrix} \delta \\ V \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_N \\ \hdashline V_1 \\ \vdots \\ V_N \end{bmatrix} \qquad f(x) = \begin{bmatrix} P(x) \\ Q(x) \end{bmatrix} = \begin{bmatrix} P_1(x) \\ \vdots \\ P_N(x) \\ \hdashline Q_1(x) \\ \vdots \\ Q_N(x) \end{bmatrix}$$

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Power Flow

$\Rightarrow$

$$y_1 = f_1(x_1, x_2, \cdots, x_n)$$
$$\vdots$$
$$y_n = f_n(x_1, x_2, \cdots, x_n)$$

Taylor series expansion around $(x_1^0, \cdots, x_n^0)$:

$$y_1 = \underbrace{f_1(x_1^0, \cdots, x_n^0)}_{y_1^0} + \left.\frac{\partial f_1}{\partial x_1}\right|_0 \Delta x_1^0 + \cdots + \left.\frac{\partial f_1}{\partial x_n}\right|_0 \Delta x_n^0$$
$$\vdots$$
$$y_n = \underbrace{f_n(x_1^0, \cdots, x_n^0)}_{y_n^0} + \left.\frac{\partial f_n}{\partial x_1}\right|_0 \Delta x_1^0 + \cdots + \left.\frac{\partial f_n}{\partial x_n}\right|_0 \Delta x_n^0$$

where

$$\Delta y_1^0 \triangleq y_1 - y_1^0, \quad \Delta x_1^0 \triangleq x_1 - x_1^0, \quad \left.\frac{\partial f_1}{\partial x_1}\right|_0 = \frac{\partial f_1}{\partial x_1}(x_1^0, \cdots, x_n^0)$$

$$\Delta y_n^0 \triangleq y_n - y_n^0, \quad \Delta x_n^0 \triangleq x_n - x_n^0, \quad \left.\frac{\partial f_n}{\partial x_n}\right|_0 = \frac{\partial f_n}{\partial x_n}(x_1^0, \cdots, x_n^0)$$

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Power Flow

where

$$\Delta y_1^0 \triangleq y_1 - y_1^0, \quad \Delta x_1^0 \triangleq x_1 - x_1^0, \quad \left.\frac{\partial f_1}{\partial x_1}\right|_0 = \frac{\partial f_1}{\partial x_1}(x_1^0, \cdots, x_n^0)$$

$$\Delta y_n^0 \triangleq y_n - y_n^0, \quad \Delta x_n^0 \triangleq x_n - x_n^0, \quad \left.\frac{\partial f_n}{\partial x_n}\right|_0 = \frac{\partial f_n}{\partial x_n}(x_1^0, \cdots, x_n^0)$$

$\Rightarrow$

$$\Delta y_1^0 = \left.\frac{\partial f_1}{\partial x_1}\right|_0 \Delta x_1^0 + \cdots + \left.\frac{\partial f_1}{\partial x_n}\right|_0 \Delta x_n^0$$

$$\vdots$$

$$\Delta y_n^0 = \left.\frac{\partial f_n}{\partial x_1}\right|_0 \Delta x_1^0 + \cdots + \left.\frac{\partial f_n}{\partial x_n}\right|_0 \Delta x_n^0$$

In matrix form,

$$\begin{bmatrix} \Delta y_1^i \\ \vdots \\ \Delta y_n^i \end{bmatrix} = \underbrace{\begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_i & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_i \\ \vdots & & \\ \left.\frac{\partial f_n}{\partial x_1}\right|_i & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_i \end{bmatrix}}_{J,\ \text{Jacobian matrix}} \begin{bmatrix} \Delta x_1^i \\ \vdots \\ \Delta x_n^i \end{bmatrix}$$

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Power Flow

In matrix form,

$$\begin{bmatrix} \Delta y_1^i \\ \vdots \\ \Delta y_n^i \end{bmatrix} = \underbrace{\begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_i & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_i \\ \vdots & & \\ \left.\frac{\partial f_n}{\partial x_1}\right|_i & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_i \end{bmatrix}}_{J,\ \text{Jacobian matrix}} \begin{bmatrix} \Delta x_1^i \\ \vdots \\ \Delta x_n^i \end{bmatrix}$$

For Power Flow Problem:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix}$$

Jacobian Matrix:

$$J = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}, \quad J_1 = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_N} \\ \vdots & & \vdots \\ \frac{\partial P_N}{\partial \delta_1} & \cdots & \frac{\partial P_N}{\partial \delta_N} \end{bmatrix}$$

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Power Flow

For Power Flow Problem:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix}$$

Jacobian Matrix:

$$J = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}, \quad J_1 = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_N} \\ \vdots & & \vdots \\ \frac{\partial P_N}{\partial \delta_1} & \cdots & \frac{\partial P_N}{\partial \delta_N} \end{bmatrix}$$

Power equations:

$$P_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

J_1: *For* $n \neq k$, *off-diagonals*

$$\frac{\partial P_k}{\partial \delta_n} = V_k Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

For $n = k$, *diagonals*

$$\frac{\partial P_k}{\partial \delta_k} = -V_k \sum_{\substack{n=1 \\ n \neq k}}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

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Power Flow

For Power Flow Problem:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V}\\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V}\end{bmatrix}\begin{bmatrix}\Delta \delta\\ \Delta V\end{bmatrix}$$

Jacobian Matrix:

$$J = \begin{bmatrix}J_1 & J_2\\ J_3 & J_4\end{bmatrix},$$

Power equations:

$$P_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

J_2: For $n \neq k$,

$$\frac{\partial P_k}{\partial V_n} = V_k Y_{kn} \cos(\delta_k - \delta_n - \theta_{kn})$$

For $n = k$,

$$\frac{\partial P_k}{\partial V_n} = \sum_{\substack{n=1\\ n\neq k}}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn}) + 2V_k Y_{kk} \cos(\delta_k - \delta_k - \theta_{kk})$$

$$= V_k Y_{kk} \cos\theta_{kk} + \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

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Power Flow

For Power Flow Problem:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V}\\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V}\end{bmatrix}\begin{bmatrix}\Delta \delta\\ \Delta V\end{bmatrix}$$

Jacobian Matrix:

$$J = \begin{bmatrix}J_1 & J_2\\ J_3 & J_4\end{bmatrix},$$

Power equations:

$$P_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

J_3: For $n \neq k$,

$$\frac{\partial Q_k}{\partial \delta_n} = -V_k Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

For $n = k$,

$$\frac{\partial Q_k}{\partial \delta_k} = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

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Power Flow

For Power Flow Problem:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V}\\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V}\end{bmatrix}\begin{bmatrix}\Delta \delta\\ \Delta V\end{bmatrix}$$

Jacobian Matrix:

$$J = \begin{bmatrix}J_1 & J_2\\ J_3 & J_4\end{bmatrix},$$

Power equations:

$$P_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

J_4: For $n \neq k$,

$$\frac{\partial Q_k}{\partial V_n} = V_k Y_{kn} \sin(\delta_k - \delta_n - \theta_{kn})$$

For $n = k$,

$$\frac{\partial Q_k}{\partial V_k} = \sum_{\substack{n=1\\ n\neq k}}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn}) + 2V_k Y_{kk} \sin(\delta_k - \delta_k - \theta_{kk})$$

$$= V_k Y_{kk} \sin\theta_{kk} + \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

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For Power Flow Problem:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V}\\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V}\end{bmatrix}\begin{bmatrix}\Delta \delta\\ \Delta V\end{bmatrix}$$

Power equations:

$$P_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

Solution Procedure: Given $x^i = \begin{bmatrix}\delta^i\\ V^i\end{bmatrix}$ at the i-th iteration

Step 1: Compute mismatch,

$$\Delta y^i = \begin{bmatrix}\Delta P^i\\ \Delta Q^i\end{bmatrix} = \begin{bmatrix}\underbrace{P}_{\text{specified}} - \underbrace{P(x^i)}_{\text{calculated}}\\ \underbrace{Q}_{\text{specified}} - \underbrace{Q(x^i)}_{\text{calculated}}\end{bmatrix}$$

Step 2: Compute Jacobian matrix J^i – updated at x^i

Jacobian Matrix: $J = \begin{bmatrix}J_1 & J_2\\ J_3 & J_4\end{bmatrix}$,

Step 3: Solve

$$J^i \Delta x^i = \Delta y^i$$

for $\Delta x^i = \begin{bmatrix}\Delta\delta^i\\ \Delta V^i\end{bmatrix}$.

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Power Flow

Step 4: Update

$$x^{i+1} = x^i + \Delta x^i$$

Repeat iterations until convergence is obtained, or the # of iterations exceeds a specified maximum.

Convergence criteria:

$$|\Delta y^i|_{max} \le \varepsilon, \quad \text{power mismatch}$$

Compute mismatch,

$$\Delta y^i = \begin{bmatrix}\Delta P^i\\ \Delta Q^i\end{bmatrix} = \begin{bmatrix}\underbrace{P}_{\text{specified}} - \underbrace{P(x^i)}_{\text{calculated}}\\ Q - Q(x^i)\end{bmatrix}$$

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Power Flow

Voltage-Controlled Bus:

V_k is specified.

⇒ Equation for Q_k is not needed.

⇒ Omit V_k & Q_k in x & y vectors.
Omit columns & rows corresponding to $\frac{\partial Q_k}{\partial V_k}$

Compute Q_k

Check $Q_{G_k} = Q_k + Q_{L_k}$ for its limits.

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Power Flow

Compute Q_k

Check $Q_{G_k} = Q_k + Q_{L_k}$ for its limits.

If it exceeds a limit, set it to the limit and

$$Q_k = Q_{G_k\,max\ or\ min} - Q_{L_k}$$

Change the bus to a load bus (P-Q)

Add corresponding rows & columns

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Power Flow

Swing Bus:

V_1 & δ_1 : given

$\Rightarrow$ Omit these variables and P_1 & Q_1

$\Rightarrow$ $k = 2, \cdots, N$

Compute P_1 & Q_1 after convergence.

Power equations:

$$P_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

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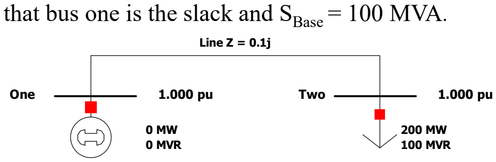
Two Bus Newton-Raphson Example

For the two-bus power system shown below, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two. Assume that bus one is the slack and S_{Base} = 100 MVA.



$$\mathbf{x} = \begin{bmatrix} \delta_2 \\ |V_2| \end{bmatrix} \qquad \mathbf{Y}_{bus} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix} = \begin{bmatrix} 10\angle -90 & 10\angle 90 \\ 10\angle 90 & 10\angle -90 \end{bmatrix}$$

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Power balance equation at bus k: $\delta_{kn} = \delta_k - \delta_n$

$$P_k = \sum_{n=1}^{N} |V_k||V_n||Y_{kn}|\cos(\delta_{kn} - \theta_{kn}) = P_{Gk} - P_{Dk}$$

$$Q_k = \sum_{k=1}^{N} |V_k||V_n||Y_{kn}|\sin(\delta_{kn} - \theta_{kn}) = Q_{Gk} - Q_{Dk}$$

Bus two power balance equations

$$P_2 = |V_2||V_1||Y_{21}|\cos(\delta_2 - \delta_1 - \theta_{21}) + |V_2|^2|Y_{22}|\cos(\delta_2 - \delta_2 - \theta_{22})$$

$$Q_2 = |V_2||V_1||Y_{21}|\sin(\delta_2 - \delta_1 - \theta_{21}) + |V_2|^2|Y_{22}|\sin(\delta_2 - \delta_2 - \theta_{22})$$

$$P_2 = |V_2||V_1|(10)\cos(\delta_2 - 0 - 90) + |V_2|^2(10)\cos(0 - (-90))$$

$$Q_2 = |V_2||V_1|(10)\sin(\delta_2 - 0 - 90) + |V_2|^2(10)\sin(0 - (-90))$$

$$|V_2||V_1|(10\sin\delta_2) + 2.0 = 0$$

$$|V_2||V_1|(-10\cos\delta_2) + |V_2|^2(10) + 1.0 = 0$$

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Two Bus Example, cont'd

General power balance equations

$$P_k = \sum_{n=1}^{N} |V_k||V_n|(G_{kn}\cos\delta_{kn} + B_{kn}\sin\theta_{kn}) = P_{Gk} - P_{Gk}$$

$$Q_k = \sum_{n=1}^{N} |V_k||V_n|(G_{kn}\sin\theta_{kn} - B_{kn}\cos\theta_{kn}) = Q_{Gk} - Q_{Gk}$$

Bus two power balance equations

$$|V_2||V_1|(10\sin\delta_2) + 2.0 = 0$$

$$|V_2||V_1|(-10\cos\delta_2) + |V_2|^2(10) + 1.0 = 0$$

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Two Bus Example, cont'd

$$P_2(\mathbf{x}) = |V_2|(10\sin\delta_2) + 2.0 = 0$$

$$Q_2(\mathbf{x}) = |V_2|(-10\cos\delta_2) + |V_2|^2(10) + 1.0 = 0$$

Now calculate the power flow Jacobian

$$J(\mathbf{x}) = \begin{bmatrix} \dfrac{\partial P_2(\mathbf{x})}{\partial \delta_2} & \dfrac{\partial P_2(\mathbf{x})}{\partial |V_2|} \\ \dfrac{\partial Q_2(\mathbf{x})}{\partial \delta_2} & \dfrac{\partial Q_2(\mathbf{x})}{\partial |V_2|} \end{bmatrix}$$

$$= \begin{bmatrix} 10|V_2|\cos\delta_2 & 10\sin\delta_2 \\ 10|V_2|\sin\delta_2 & -10\cos\delta_2 + 20|V_2| \end{bmatrix}$$

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Two Bus Example, First Iteration

Set $v = 0$, guess $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Calculate

$$\mathrm{f}(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10\sin\delta_2) + 2.0 \\ |V_2|(-10\cos\delta_2) + |V_2|^2(10) + 1.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10|V_2|\cos\delta_2 & 10\sin\delta_2 \\ 10|V_2|\sin\delta_2 & -10\cos\delta_2 + 20|V_2| \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

Solve $\mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}$

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Two Bus Example, Next Iterations

$$\mathrm{f}(\mathbf{x}^{(1)}) = \begin{bmatrix} 0.9(10\sin(-0.2)) + 2.0 \\ 0.9(-10\cos(-0.2)) + 0.9^2 \times 10 + 1.0 \end{bmatrix} = \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(1)}) = \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}$$

$$\mathbf{x}^{(2)} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} - \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}^{-1} \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix} = \begin{bmatrix} -0.233 \\ 0.8586 \end{bmatrix}$$

$$\mathrm{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 0.0145 \\ 0.0190 \end{bmatrix} \qquad \mathbf{x}^{(3)} = \begin{bmatrix} -0.236 \\ 0.8554 \end{bmatrix}$$

$$\mathrm{f}(\mathbf{x}^{(3)}) = \begin{bmatrix} 0.0000906 \\ 0.0001175 \end{bmatrix} \qquad \text{Done!} \qquad V_2 = 0.8554\angle -13.52^\circ$$

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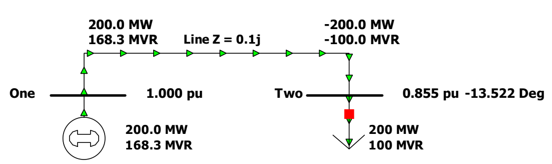
Two Bus Solved Values

Once the voltage angle and magnitude at bus 2 are known we can calculate all the other system values, such as the line flows and the generator reactive power output



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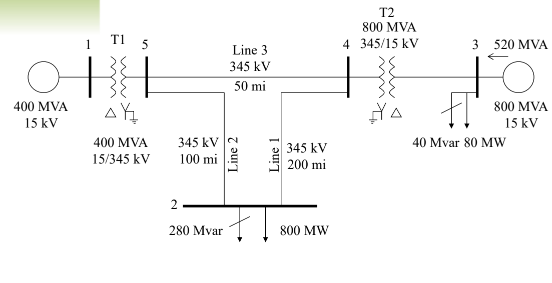
The N-R Power Flow: 5-bus Example



Single-line diagram

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The N-R Power Flow: 5-bus Example

Table 1. Bus input data

Bus	Type	V per unit	δ degrees	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit	Q_{Gmax} per unit	Q_{Gmin} per unit
1	Swing	1.0	0	—	—	0	0	—	—
2	Load	—	—	0	0	8.0	2.8	—	—
3	Constant voltage	1.05	—	5.2	—	0.8	0.4	4.0	-2.8
4	Load	—	—	0	0	0	0	—	—
5	Load	—	—	0	0	0	0	—	—

Table 2. Line input data

Bus-to-Bus	R' per unit	X' per unit	G' per unit	B' per unit	Maximum MVA per unit
2-4	0.0090	0.100	0	1.72	12.0
2-5	0.0045	0.050	0	0.88	12.0
4-5	0.00225	0.025	0	0.44	12.0

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The N-R Power Flow: 5-bus Example

Table 3. Transformer input data

Bus-to-Bus	R per unit	X per unit	G_c per unit	B_m per unit	Maximum MVA per unit	Maximum TAP Setting per unit
1-5	0.00150	0.02	0	0	6.0	—
3-4	0.00075	0.01	0	0	10.0	—

Table 4. Input data and unknowns

Bus	Input Data	Unknowns
1	$V_1 = 1.0$, $\delta_1 = 0$	P_1, Q_1
2	$P_2 = P_{G2}-P_{L2} = -8$ $Q_2 = Q_{G2}-Q_{L2} = -2.8$	V_2, δ_2
3	$V_3 = 1.05$ $P_3 = P_{G3}-P_{L3} = 4.4$	Q_3, δ_3
4	$P_4 = 0$, $Q_4 = 0$	V_4, δ_4
5	$P_5 = 0$, $Q_5 = 0$	V_5, δ_5

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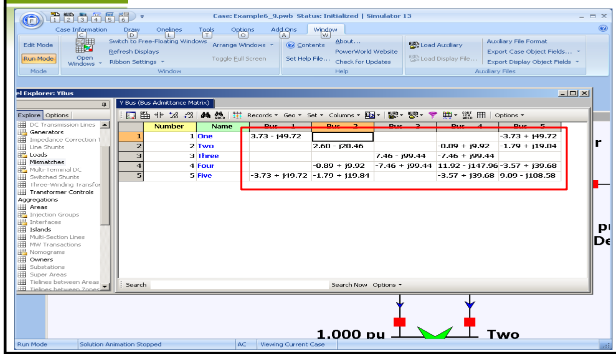
Time to Close the Hood: Let the Computer Do the Math! (Ybus Shown)



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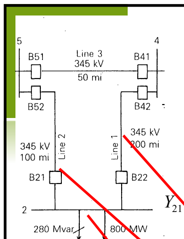

Bus-to-Bus	R′ per unit	X′ per unit	G′ per unit	B′ per unit
2–4	0.0090	0.100	0	1.72
2–5	0.0045	0.050	0	0.88
4–5	0.00225	0.025	0	0.44

Elements of Y_{bus} connected to bus 2

$$Y_{21} = Y_{23} = 0$$

$$Y_{24} = \frac{-1}{R'_{24} + jX'_{24}} = \frac{-1}{0.009 + j0.1} = -0.89276 + j9.91964 \ per\ unit$$

$$Y_{25} = \frac{-1}{R'_{25} + jX'_{25}} = \frac{-1}{0.0045 + j0.05} = -1.78552 + j19.83932 \ per\ unit$$

$$Y_{22} = \frac{1}{R'_{24} + jX'_{24}} + \frac{1}{R'_{25} + jX'_{25}} + j\frac{B'_{24}}{2} + j\frac{B'_{25}}{2}$$

$$= (0.89276 - j9.91964) + (1.78552 - j19.83932) + j\frac{1.72}{2} + j\frac{0.88}{2}$$

$$= 2.67828 - j28.4590 = 28.5847\angle -84.624° \ per\ unit$$

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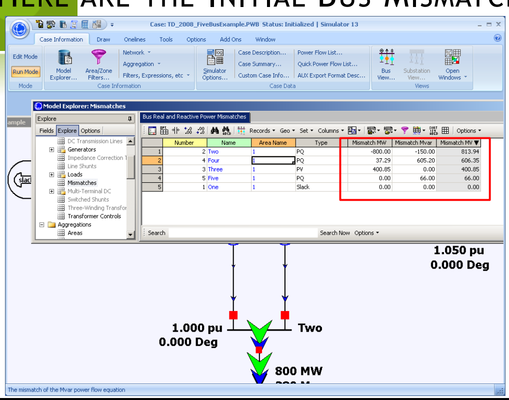
Here are the Initial Bus Mismatches



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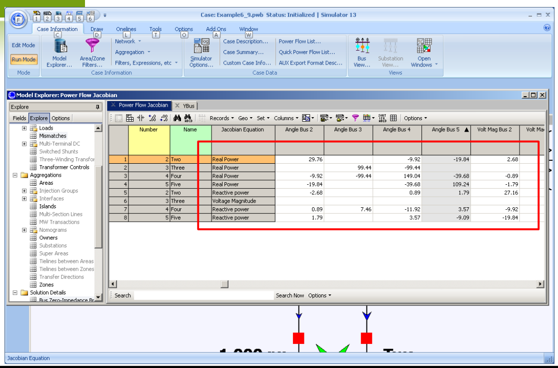
And the Initial Power Flow Jacobian



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And the Hand Calculation Details!

$$\Delta P_2(0) = P_2 - P_2(x) = P_2 - V_2(0)\{Y_{21}V_1\cos[\delta_2(0)-\delta_1(0)-\theta_{21}]$$
$$+Y_{22}V_2\cos[-\theta_{22}] + Y_{23}V_3\cos[\delta_2(0)-\delta_3(0)-\theta_{23}]$$
$$+Y_{24}V_4\cos[\delta_2(0)-\delta_4(0)-\theta_{24}]$$
$$+Y_{25}V_5\cos[\delta_2(0)-\delta_5(0)-\theta_{25}]\}$$
$$= -8.0 - 1.0\{28.5847(1.0)\cos(84.624°)$$
$$+9.95972(1.0)\cos(-95.143°)$$
$$+19.9159(1.0)\cos(-95.143°)\}$$
$$= -8.0 - (-2.89\times10^{-4}) = -7.99972 \ per\ unit$$

$$J1_{24}(0) = V_2(0)Y_{24}V_4(0)\sin[\delta_2(0)-\delta_4(0)-\theta_{24}]$$
$$= (1.0)(9.95972)(1.0)\sin[-95.143°]$$
$$= -9.91964 \ per\ unit$$

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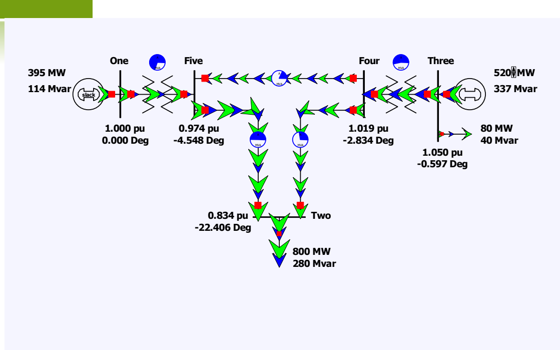
Five Bus Power System Solved



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Solving Large Power Systems

- The most difficult computational task is inverting the Jacobian matrix
  - inverting a full matrix is an order N^3 operation, meaning the amount of computation increases with the cube of the size
  - this amount of computation can be decreased substantially by recognizing that since the Y_{bus} is a *sparse matrix*, the Jacobian is also a sparse matrix
  - using sparse matrix methods results in a computational order of about $N^{1.5}$.
  - this is a substantial savings when solving systems with tens of thousands of buses

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Newton-Raphson Power Flow

- Advantages
  - fast convergence as long as initial guess is close to solution
  - large region of convergence
- Disadvantages
  - each iteration takes much longer than a Gauss-Seidel iteration
  - more complicated to code, particularly when implementing sparse matrix algorithms
- Newton-Raphson algorithm is very common in power flow analysis

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